JOURNAL OF ALGEBRA 24, 613 (1973)

Erratum

Volume 23, No. 1 (1972), in the article, "Solvability of Finite Groups Admitting Fixed-Point-Free Automorphisms of Orders rs," by Elizabeth Wall Ralston, pp. 164–180. The following should come directly before the acknowledgments on page 179:

So we assume $O_t(M) = 1$. Since M is of odd order, the $Z(J)$-theorem of Glauberman implies $M = N_M(Z(J(M_p)))$. The maximality of M means that $M = [N(Z(J(M_p)))]_{p,t} \supseteq N_P(Z(J(M_p)))$. Since $Z(J(M_p))$ char M_p, $N_P(M_p) \subseteq M$. Hence, $M_p = P$ and $PT = TP$.

A parallel argument shows that $PQ = QP$ for $p \in \gamma$ and $q \in \tau$, p and q odd. Hence, $\Gamma_{2'}$, $\Theta_{2'}$ and $\Gamma_{2'}$, $K_{2'}$, are solvable σ-invariant Hall subgroups of G. From this it follows that if $2 \notin \gamma$, then G contains a solvable σ-invariant Hall $\gamma \cup \tau$- or $\gamma \cup \kappa$-subgroup, and we are done.

Thus, we may assume $2 \in \gamma$. Let $S = G_2$. Suppose $Z(S) \neq Z(S)_1$. Without loss we may assume $Z(S)_\phi \neq 1$. Then for $t \in \tau$, $[N(Z(S)_\phi)]_{2,t} = TS = ST$ by Theorem 4.2. Thus $\Gamma\Theta = \Theta\Gamma$ is a solvable σ-invariant Hall subgroup of G.

Finally we assume that $2 \in \gamma$ and $Z(S) = Z(S)_1$. Without loss we assume $3 \notin \tau$. We shall now show that $ST = TS$ for all $t \in \tau$.

Pick $t \in \tau$ and let M be a maximal σ-invariant $\{2, t\}$-subgroup of G containing $[N(S_\phi)]_{2,t} \supseteq T, Z(S)$. Then $Z(S) \subseteq O_2(M)$ by Lemma 3.5. If $O_t(M) \neq 1$, then $M \supseteq [N(Z(S))]_{2,t} \supseteq S$ by Lemma 5.4. Hence, $TS = ST$.

Thus, we assume $O_t(M) = 1$. Let $\bar{S}$ be the σ-invariant S_2-subgroup of M. Since S_4 is not involved in M, $M = \langle C_M(Z(\bar{S})), N_M(J(\bar{S}))\rangle$ by Corollary 10 of [7]. Then $T = \langle C_T(Z(\bar{S})), N_T(J(\bar{S}))\rangle$. Since $Z(S) \subseteq Z(\bar{S})$, $C_T(Z(\bar{S})) \subseteq C_T(Z(S)) = \bar{T}$, where $\bar{T}$ is permutable with S (in $C(Z(S))$). Now let $\hat{S} \supseteq \bar{S}$ be a maximal σ-invariant 2-subgroup of G such that $\langle \bar{T}, \hat{T}\rangle = T$, where $\hat{T} = N_T(J(\hat{S}))$. Let $\hat{M}$ be a maximal σ-invariant $\{2, t\}$-subgroup of G containing $[N(J(\hat{S}))]_{2,t} \supseteq \hat{T}$. $Z(S) \subseteq O_2(\hat{M})$ by Lemma 3.5. If $O_t(M) \neq 1$, then $S \subseteq [N(Z(S))]_{2,t} \subseteq \hat{M}$ by Lemma 5.4. In that case S is permutable with $\hat{M}_t \supseteq \hat{T}$. It follows that S is permutable with $\langle \hat{M}_t, \bar{T}\rangle = T$.

Thus we may assume that $O_t(\hat{M}) = 1$. Hence, $\hat{M} = \langle C_{\hat{M}}(Z(\tilde{S})), N_{\hat{M}}(J(\tilde{S}))\rangle$ and $\hat{M}_t = \langle C_{\hat{M}_t}(Z(\tilde{S})), N_{\hat{M}_t}(J(\tilde{S}))\rangle$, where $\tilde{S} = \hat{M}_2$. But $Z(S) \subseteq Z(\tilde{S})$, so $C_{\hat{M}_t}(Z(\tilde{S})) \subseteq C_T(Z(S)) = \bar{T}$. Since $\hat{T} \subseteq \hat{M}_t$, $T = \langle \bar{T}, \hat{M}_t\rangle = \langle \bar{T}, N_T(J(\tilde{S}))\rangle$. Now $\tilde{S} \supseteq \hat{S}$, so the maximality of $\hat{S}$ implies that $\tilde{S} = \hat{S}$. However, $\tilde{S} \supseteq [N(J(\hat{S}))]_{2,t} \cap S = N_S(J(\hat{S}))$. But $N_S(J(\hat{S})) \supsetneq \hat{S}$ unless $S = \hat{S}$. We conclude that $\hat{S} = S$ is permutable with $\langle \bar{T}, \hat{T}\rangle = T$, and we are done.